# VEHICLE PARK MANAGEMENT THROUGH THE GOAL PROGRAMMING MODEL

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## ABSTRACT

The "Ministère des transports du Québec" has a fleet of equipment to maintain the roads of the province of Quebec. The management of the fleet is the responsibility of the "Centre de Gestion de l'équipement Roulant". They decide whether an old item of equipment will be sold, replaced or maintained. Several factors, sometimes conflicting, come into play in the decision making process. They include reducing the fleet by 20%, respecting the budget, meeting the customer requirements, and maintaining and renovating certain aspects of the equipment. As a management tool, we propose Goal Programming model with the integration of the decision-maker's preferences.

**Keywords:** Vehicle Park Management, Goal Programming Model, Managers' Preferences, Satisfaction Functions.

## RÉSUMÉ

Le Ministère des Transports du Québec dispose d'une flotte d'équipements pour l'entretien des routes de la province du Québec. L'unité responsable de la gestion de ce parc est le Centre de Gestion de l'équipement Roulant qui doit décider pour chaque unité ou équipement soit de continuer à l'entretenir, le vendre ou le remplacer. Plusieurs facteurs, souvent conflictuels, sont à considérer lors du processus décisionnel tels que : réduire la flotte de 20%, respecter l'enveloppe budgétaire, répondre aux besoins des clients ou encore maintenir un certain rajeunissement du parc. Dans le présent document, nous proposons comme outil de gestion le modèle du "Goal Programming" avec l'intégration des préférences du gestionnaire.

**Mots clés:** la gestion du parc des équipements roulants, le modèle du Goal Programming, les préférences des gestionnaires, les fonctions de satisfaction.

## 1. INTRODUCTION

In 1995, the "Ministère des transports du Québec" (MQT) created a new autonomous unit, the "Centre de Gestion de l'équipement Roulant" (CGER). The CGER is responsible for the management of the MTQ vehicles park. The CGER mission is to ensure the availability of the vehicles and the related equipment for the customers and the maintenance of vehicles to keep them in good operating condition (Gouvernement du Québec, 1996; Gouvernement du Québec, Ministère des Transports, 1997). The current equipment park of the Ministry is composed of 1450 light vehicles, 650 trucks, 1000 motorised pieces of equipment, and almost 5000 other equipments. The equipment in this park is particularly used for maintenance of the road network. The units are stored in categories with considerable size variation among categories. There are 26 categories, but data are available for 11 categories. The CGER assumes almost all the costs associated with the park such as preventive maintenance, minor and major

Recd. Oct. 2001, Rev. Oct. 2002, Acc. Jan. 2003

repairs, and the costs of purchase or disposal of vehicles. Within a budgetary restriction, the greatest challenge for the CGER is to find a way of managing the park at a low cost while offering a good service to its customers. The managers face a multitude of constraints. They have to take into consideration the budgetary constraint, the fact that the categories do not all have the same relative importance and consider customer needs with respect to the number of units in each category while renovating the equipment park. It is also necessary to add to this last constraint, the desire of the government to reduce the park by 20% during the next year.

The aim of this paper is to propose an effective model for the vehicle park management that respects the government's budgetary constraints, the customers' needs and the MTQ managers' preferences.

## 2. PROBLEM FORMULATION

To achieve the goal of the CGER, we had to decide whether each unit in the park should be sold, replaced or maintained. This leads us to choose between a multitude of possible combinations while taking into account the optimisation of several objectives simultaneously. In our formulation, we consider two aspects of the park management. First, the government's desire to reduce the fleet by 20%, and second the needs of the customers (53 service centres of the MTQ) that are not compatible with the 20% reduction. Given the circumstance, the Goal Programming model (GP) was an appropriate management tool for this problem. The GP, an easy model to understand and to use, is in fact an extension of linear programming that has powerful solution algorithms. First developed by Charnes *et al.* (1955) and Charnes and Cooper (1961) then applied by Lee (1973) and Lee and Clayton (1972), the GP model gained a great deal of popularity and its use has spread in diversified fields such as: management of water basins, management of solid waste, accounting and financial aspect of stock management, marketing, quality control, human resources, production, transportation and site selection, space studies, telecommunications, agriculture, forestry and aviation (Aouni and Kettani, 2001). The weighted GP model has the following form:

$$\begin{aligned} \text{Program M1:} \quad & \text{Min. } Z = \textstyle\sum_{l=1}^{p} (w_l^+ \delta_l^+ + w_l^- \delta_l^-) \\ \text{subject to:} \quad & \textstyle\sum_{k=1}^{K} a_{lk} x_k + \delta_l^- - \delta_l^+ = b_l, \ \forall l \\ & x \in F, \\ & \delta_l^+, \delta_l^- \geq 0 \ (\text{for } l = 1, 2, \ldots, p), \end{aligned}$$

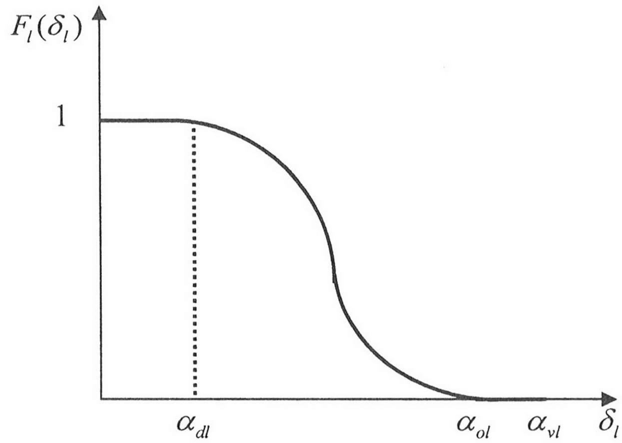
where:

$x_k$: decision variables,
$b_l$: goal associated with objective $l$,
$a_k$: technological parameters,
$\delta_l^+, \delta_l^-$: positive and negative deviations associated with objective $l$,
$w_l^+, w_l^-$: coefficients of relative importance associated to the positive and negative deviations respectively,
$F$: feasible solution set.

The coefficients $w_l^+$ and $w_l^-$ are applied to introduce partially the decision-maker's (DM) preferences. However, the DM's preferences can be expressed explicitly through the concept of satisfaction functions (Martel and Aouni, 1990). The general form of the satisfaction function $F_l(b_l)$ is as follows (Fig. 1):

The functions $F_l(\delta_l)$ allow measuring the DM (or the manager) satisfaction according to the deviations $\delta_l$ associated to the goals $b_l$. The satisfaction functions can vary from one DM to another. As a starting point, Martel and Aouni (1990) took the generalized criteria of the PROMETHEE method (Brans *et al.*, 1984) to define various forms of satisfaction function. We note that this function has three thresholds: a) indifference threshold $\alpha_{dl}$: all solutions

**Figure 1:** The standard form of satisfaction function




with a deviation lower than threshold $\alpha_{dl}$ result in maximum satisfaction for the DM, b) null satisfaction threshold $\alpha_{ol}$: the satisfaction of the decision maker is null if the deviation reaches this value, and c) *veto* threshold $\alpha_{vl}$: any solution with a deviation higher than this threshold is rejected. With the introduction of such satisfaction functions, the objective function and the constraints of the preceding formulation can be written in the following form:

$$\begin{aligned}
&\text{Program M2:} \quad \text{Max. } Z = \textstyle\sum_{l=1}^{p} (w_l^+ F_l^+(\delta_l^+) + w_l^- F_l^-(\delta_l^-)) \\
&\text{subject to:} \quad \textstyle\sum_{k=1}^{K} a_{lk}x_k + \delta_l^- - \delta_l^+ = b_l, \ \forall l \\
&\qquad x \in F, \\
&\qquad 0 \le \delta_l^+, \le \alpha_{vl}^+ \text{ and } 0 \le \delta_l^- \le \alpha_{vl}^-, \ (\text{for } l = 1, 2, \ldots, p),
\end{aligned}$$

## 2.1 First Government Objective

Since the government wished to reduce the vehicles in the park by 20% during the next year, our objective was to determine a fleet combination that satisfied this objective. In this first stage, we also took into account the preferences of the managers, like the intervention to take regarding each unit or the relative importance of each category of units. For example, as the government wanted to renovate the fleet, they gave more weight to the replacement of the units. To consider the government's objective, we proposed the following program:

$$\begin{aligned}
&\text{Program M3:} \quad \text{Max. } S = \textstyle\sum_{i=1}^{I} \sum_{j=1}^{n_i} \sum_{k=1}^{3} c_k w_i x_{ijk} \\
&\text{subject to:} \quad \textstyle\sum_{j=1}^{n_i} x_{ij1} + \sum_{j=1}^{n_i} x_{ij3} \le g_i, \\
&\qquad \textstyle\sum_{k=1}^{3} x_{ijk} = 1, \\
&\qquad x_{ijk} = \{0, 1\} \ (\text{for } i = 1, 2, \ldots, I \text{ and } j = 1, 2, \ldots, n_i),
\end{aligned}$$

where:

$x_{ijk}$: the unit $j$ of category $i$ undergoing the intervention $k (k = 1, 2, 3)$,

$$x_{ij1} = \begin{cases} 1, & \text{maintain the unit } j \text{ of the category } i \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ij2} = \begin{cases} 1, & \text{sell the unit } j \text{ of the category } i \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ij3} = \begin{cases} 1, & \text{replace the unit } j \text{ of the category } i \\ 0, & \text{otherwise.} \end{cases}$$

$i = 1, 2, \ldots, I$ (categories), $j = 1, 2, \ldots, n_i$ ($n_i$ is the number of units in category $i$),
$c_k$: the importance associated with the intervention $k$ as expressed by the management,
$w_i$: the importance associated with the category $i$ as expressed by the management,
$g_i$: the number of units that the government wanted (reduction of 20%) for category $i$.

Note that under the 20% fleet reduction constraint, we do not retain the case where $k = 2$ (sell a unit) since they are units to be sold, therefore, they are not included in the new fleet, unlike the unit to maintain ($k = 1$) and the unit to replace ($k = 3$) which form the new fleet. The optimal value of the objective function S* (obtained through the resolution of the program M3) was introduced into the GP model as a goal.

## 2.2 Main Model

The main model was a multiple objective mathematical program composed of three objectives for which goals are fixed. This model contained constraints (associated with each objective) expressed in the form of equalities using the positive and negative deviations. The solution generated by this model represented the best compromise fleet in the park of the MTQ. This solution has taken into account the objectives of the government as well as the customer requirements so that the level of satisfaction of the manager was as high as possible. Maximization of the satisfaction level constituted the objective function of this main program. There were three kinds of constraints included in this model. The constraints are as follows:

**The 20% reduction constraint**

If S* cannot be attained, the positive and negative deviations allow the program to measure how far a solution was from S* (value obtained from the program M3). It should be noted that one expected negative deviation, if any, since the value S* represented, to some extent, an ideal point.

**The budgetary constraint**

The MTQ allocates a budget for the operation and management of its park during each fiscal year. The managers of the CGER provided us the data regarding the maintenance costs, the sales prices as well as the unit purchase prices for each equipment category. This constraint considered only the positive deviation, which means that a certain amount of over budget was permitted. The negative deviation was not retained because spending less than the budget could have adverse consequences on the future budgets.

**The customers' needs constraint**

This constraint allowed the program to sustain the number of units in each category to a level assessed as the customers needs. It should be noted that the expressed requirements were in general higher than the government policy permitted (20% reduction). This clearly indicated the conflict of objectives included in this program since the constraint S* was obtained with a reduction of the fleet by 20%, in addition to the budgetary constraint which also restricted the range of solutions whereas the constraint of the needs tended to increase the fleet size.

The main program took the following form:

Program M4: Max. $Z = F_S^-(\delta_S^-) + F_S^+(\delta_S^+) + F_B^+(\delta_B^+) + \sum_{i=1}^{I}(F_i^-(\delta_{bi}^-) + F_i^+(\delta_{bi}^+))$

subject to:

constraint on S*

$$\sum_{i=1}^{I} \sum_{j=1}^{n_i} \sum_{k=1}^{3} c_k w_i x_{ijk} + \delta_s^- - \delta_s^+ = S^*,$$

constraint on the budget B

$$\sum_{i=1}^{I} \sum_{j=1}^{n_i} \sum_{k=1}^{3} a_{ijk} x_{ijk} + \sum_{i=1}^{I} p_i y_i - \delta_B^+ = B,$$

**Table 1:** Relative weights of categories

| Category | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $w_i$ | 0.1 | 0.1 | 0.133 | 0.133 | 0.1 | 0.1 | 0.033 | 0.066 | 0.066 | 0.033 | 0.133 |

**Table 2:** Relative weights of interventions

| Interventions | Maintain | Sell | Replace |
|---|---|---|---|
| $c_k$ | 0.31 | 0.23 | 0.46 |

constraints on the needs

$$\sum_{j=1}^{ni} x_{ij1} + \sum_{j=1}^{ni} x_{ij3} + y_i + \delta_{bi}^- - \delta_{bi}^+ = b_i,\ \forall i,$$
$$\sum_{k=1}^{3} x_{ijk} = 1,$$
$$x_{ijk} = \{0,1\}\ (\text{for } i = 1,2,\ldots,I,\ j = 1,2,\ldots,n_i \text{ and } k = 1,2,3),$$
$$y_i \geq 0 \text{ and integer } \forall i,$$
$$\delta_s^+, \delta_s^-, \delta_B^+ \geq 0,\ \delta_{bi}^+, \delta_{bi}^- \geq 0\ \forall i,$$

where:

- $y_i$: number of new units added to category $i$,
- $b_i$: requirements in unit for category $i$ expressed by customers,
- $B$: total budget allocated,
- $p_i$: purchase price of a new unit of category $i$,
- $a_{ijk}$: cost of the intervention $k$ for the unit $j$ of category $i$,
- $\delta_S^+, \delta_S^-$: positive and negative deviation from $S^*$,
- $\delta_B^+$: positive deviation regarding the budget,
- $\delta_{bi}^+, \delta_{bi}^-$: positive and negative deviations of the needs, expressed in units, for category $i$.

It should be noted that a yi variable is introduced into the main program in case that the needs are higher than the current availability. It thus pure purchases of new equipments added units to the categories that required an increase. The tables 1 and 2 contain the relative weights the different vehicles categories and the three types of interventions, respectively.

## 3. SOLUTION PROCEDURE

The $c_k$ and $w_i$ parameters of the main program M4 were obtained directly from one manager of the CGER. We asked him to make a ranking (with possibility of tied ranks) of the categories. Then, he provided us with a chart where all the categories were compared with a reference category (the least important category) while indicating how many times (X time) each category is more important than the reference category. We have, in fact, proceeded, as in the SMART method (Edwards, 1977), determining the relative weights of each category (Table 1). The interventions (interventions $k = 1, 2, 3$) were also weighted but in a simpler manner. The manager directly allocated a relative importance to each intervention. We standardized them on a scale from 0 to 1 (Table 2).

To represent his preferences, the manager has retained the following simple satisfaction function (Fig. 2).

**The budgetary deviation**

As mentioned above, only the positive deviation was considered in this case: $\alpha_{ol} = 5\%$ of the value of the budget, and $\alpha_{vl} = 10\%$ (veto threshold) of the value of the budget. The satisfaction function associated to the budgetary deviation is as follows (Fig. 3):

**Figure 2:** The manager's satisfaction function

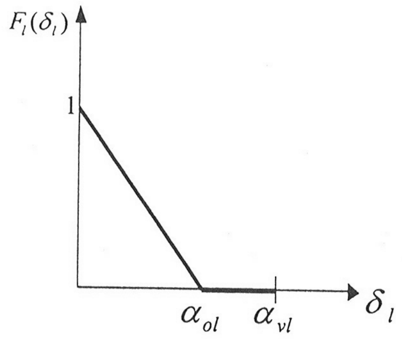
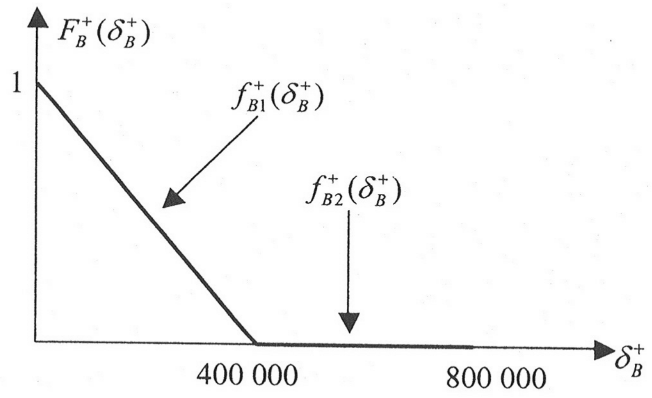



**Figure 3:** The satisfaction function of the positive budgetary deviations (budget of $8,000,000)




**The 20% reduction deviation**

We integrated the two deviations, the positive and the negative, despite the fact that the positive deviation is improbable. Indeed, exceeding the value of S* (an ideal point) would have been surprising. The fact of integrating the positive deviation in the constraint did help us to test the reliability of our program. The values of the thresholds $\alpha_{ol}$ and $\alpha_{vl}$ were subjectively fixed by the manager in a way that reflected his preferences.

**The customers' needs deviation**

To take into account the fact that the needs expressed by the customers were, in general, higher than the government's objective (reduction of 20%) and lower than the current state of the park, we chose *veto* thresholds which is a better representation of these conditions. In fact, for the negative deviations, we fixed, as *veto* threshold $\alpha_{vl}^-$, the difference between the expressed needs and the objective of the government. Thus, the program could not generate solutions with a fleet reduction of more than 20%. Concerning the dissatisfaction thresholds $\alpha_{ol}^-$, we fixed it at a value half way between the origin (0) and the veto threshold. The least weighted category received the highest value of $\alpha_{ol}$. For the other categories, the distance between the origin and the value $\alpha_{ol}$ was proportional to the relative importance of their category compared to the least important category. This means that the manager penalized more quickly (more steep line slope) the deviations on the most significant categories. Concerning the positive deviations, the *veto* threshold $\alpha_{vl}^+$, corresponded to the difference between the needs expressed for each

**Table 3:** Values of threshold of the other satisfaction functions

| | | $\delta_S$ | $\delta_1$ | $\delta_2$ | $\delta_3$ | $\delta_4$ | $\delta_5$ | $\delta_6$ | $\delta_7$ | $\delta_8$ | $\delta_9$ | $\delta_{10}$ | $\delta_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\delta_l^+$ | $\alpha_{ol}^+$ | 40 | 2 | 2 | 8 | 4 | 1 | 2 | 3 | 1 | 1 | 5 | 1 |
| | $\alpha_{vl}^+$ | 80 | 14 | 14 | 117 | 66 | 12 | 13 | 9 | 3 | 3 | 10 | 9 |
| $\delta_l^-$ | $\alpha_{ol}^+$ | 40 | 2 | 4 | 3 | 3 | 1 | 1 | 2 | 1 | 1 | 6 | 3 |
| | $\alpha_{ol}^-$ | 600 | 14 | 42 | 39 | 44 | 9 | 6 | 8 | 3 | 4 | 10 | 40 |

category and the current state. The values of the thresholds $\alpha_{vl}^+$ were selected according to the same principle as for the negative deviations. We gathered, in the Table 3, values of these thresholds.

The different satisfaction functions were introduced into the program M4. The program obtained it is nonlinear. The Oral and Kettani (1992) linearization procedure was used to obtain an equivalent linear representation of the program M4. The details of the linearization procedure are available in appendix A of this paper. The resulting linear program was solved by using the General Algebraic Modeling System. It's a high-level modeling system which integrates solvers that can deal with large scale optimisation problems.

## 4. COMPUTATIONAL RESULTS

At first, we solved the mathematical program that produces a weighted number of units by considering only the government policy (a reduction of the fleet by 20%). The value obtained ($S^* = 117.35$) was introduced as a goal in the main program. This program was solved with various budgets and with the needs expressed by the customers. The treated fleet was composed of 2459 units that was higher than the customer requirements (2189 units). The program generated the best compromise fleet with a value of the satisfaction function of (12.32) on a possible maximum of 13. This means that the level of satisfaction reached for our 13 objectives (11 objectives on the needs according to categories, one objective on the value of $S^*$, and one objective on the budget) is 94.76%. The gap to 100% of satisfaction is explained in the following way: the negative deviation of $S^*$ is $\delta_S^- = 26.87$ compared to a threshold $\alpha_{oS}^- = 40$, then a loss of satisfaction of about 26.87/40 = 0.67. The positive deviation of the budget is $\delta_B^+ = \$65.05$ compared to a threshold $\alpha_o = \$400,000$, which gave a loss of satisfaction of about 65.05/400,000 = 0.000162. All other deviations were null. This means that the fleet generated exactly met the customer's requirements.

The generated fleet was, by and large, made up of units to be maintained. The percentage of units replaced was 11.83%. Among the replaced units, the most important categories had a higher number replaced (categories 3, 4 and 11). One of the objectives of the manager was then achieved (renovate the fleet by giving priority replacing the most important categories). We made simulation runs to determine the composition of the fleet, the level of satisfaction reached and the percentage of replacement of the fleet while varying the budget. The budgets used were: 2, 3, 4, 5, 8, 10 and 12 million dollars. The results obtained are presented in table 4.

We noticed that the value of the objective function (Z) increased with the increase in the budget. With more budget, the program tends to make more replacement of units (which is desired by the managers) and thus reduced the deviation associated with the goal $S^*$. These replacements were made, primarily, in the most important categories. Like Z, the percentage of replacement, also increased with the budget. It went from 0.129% for a budget of \$2 millions to 33.89% for \$12 millions budget. We also noticed the appearance of a threshold from which the customer's requirements started to be unsatisfied at 100%. In fact, during our compilations, the

**Table 4:** Results of simulations (principal variables)

| Budgets ($\$10^6$)/Variables | 2 | 3 | 4 | 5 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| $y_i$, $i = 1,2 \ldots, 11$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\delta_S^-$ | 34.21 | 33.50 | 31.36 | 28.31 | 26.87 | 22.26 | 18.92 |
| $\delta_S^+$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\delta_B^+$ | 48 | 93 | 2 | 1984 | 65 | 329 | 147 |
| $\delta_{I1}^- = \delta_{I2}^-$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\delta_{I3}^-$ | 39 | 39 | 0 | 0 | 0 | 0 | 0 |
| $\delta_{I4}^-$ | 44 | 44 | 44 | 44 | 0 | 0 | 0 |
| $\delta_{I5}^-$ | 9 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\delta_{I6}^- = \delta_{I7}^- = \delta_{I8}^- = \delta_{I9}^- = \delta_{I10}^-$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\delta_{I11}^-$ | 40 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\delta_{Ii}^+$, $i = 1,2 \ldots, 11$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $Z$ | 7.66 | 9.53 | 11.203 | 11.28 | 12.32 | 12.44 | 12.52 |
| % of replacement | 0.129 | 0.61 | 4.43 | 11.65 | 11.83 | 24.21 | 33.89 |

negative deviations of the customer's requirements appeared with low budgets. As we increased the budget, these deviations went down until they disappeared completely with a budget of $8 millions. One of the strongest aspects of this program is its flexibility and its adaptability to all new situations. It is much appreciated tool for the managers, especially since the long-term policies undergo many changes before they are finalized. Indeed, this program allows the managers of the CGER to propose management policies according to different situations. Under normal conditions (*i.e.* conditions where the government allocates a normal budget), they can aim to achieve their long-term objectives such as the renovation of their fleet or meeting the needs of their customers. Whereas if any emergency situation or crisis arises (example: budget cuts), they can give up their long-term plan and propose a fleet according to the current situation. We simulated such a situation by assuming a budget of $2 millions, which is far under what the managers of the CGER should normally expect. The results obtained reflected this situation: rates of replacement of 0.129% (almost no replacement, maintenance only), the customer's requirements are not reached, $Z$ has a value of 7.66, which corresponds to a 58% rate of satisfaction. However, this is a situation that should not last for a long time. The consequences of such a budget would be felt in the long run. Indeed, if the fleet is not renewed regularly, its resale value drops considerably and the maintenance costs also increase as the units age. The program also enables us to determine the limits that we cannot exceed even in emergency situations. According to our simulation, we can state that this limit is a budget of around $2 millions. Indeed, with the same constraints and a budget of $1.5 million the program does not have a feasible solution. This means that we will not be able to meet the customer's requirements. Therefore, we expect fewer services to the population and less maintenance of the roads. The flexibility of the program can be extended, not only to a simple change in the value of the budget, but also to all new data, policy or orientation of the managers of the CGER. We also carried out other simulation runs to check the sensitivity of the program to the changes in other parameters. For instance, we changed the value of $\alpha_o$ for the positive deviation of the budget. We noticed that, in general, this does not have a great effect on the results. However, it appeared in other simulation runs that changing the level of tolerance of a satisfaction function influences the results. It is thus very important to pay attention to the choice of the tolerance level. One should make sure that they are most realistic.

## 5. CONCLUSION

In the course of conducting this research, we tried to be as near to reality as was possible despite the lack of some information. However, we believe that the proposed model can treat a case with complete data. Indeed, as we specified, this model can react to any small or major changes in the situation that the managers encounter. Our first objective was to find a fleet that would correspond to the requirements of the managers and the customers while respecting the budget constraint. However, while performing the simulation runs, we came up with some other points useful for the managers: a) the possibility to make decisions in situation of the budget crisis ($2 millions), b) the existence of a threshold of the budget under which we can not satisfy customer's request ($5 million), c) the existence of another threshold from which the service start to show some weakness, *i.e.* that the CGER can not accomplish any more its mission ($1.5 million). The choice of thresholds $\alpha_o$ is an important aspect of modelling. Their value must be realistic to ensure good results. There is also a very important element that was brought out during our simulations which was the possibility of controlling the percentage of renewal of the fleet according to the budget. Considering all these elements we can say that, with this model, the managers can define their best compromise fleet as well as testing new elements for future considerations.

## ACKNOWLEDGEMENTS

The authors would like to express their gratitude to the anonymous referees and Dr. Ozhand Ganjavi for their remarks and comments which have helped to improve this paper. We would like also to acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada and Laurentian University Research Fund.

## APPENDIX A

This appendix contains the details regarding the linearization procedure used section 3 of this paper.

Introduction of integer and binary variables resulted in non-linear functions in the program. To deal with this situation, we used the technique of linearization developed by Oral and Kettani (1992). This technique permits us to find an equivalent linear form for our non-linear functions. In its general form, the model proposed by Oral and Kettani is as follows:

$$\text{Min. } \textstyle\sum_{i=1}^{n}(D_i^- x_i + \xi_i)$$

subject to:

$$\xi_i \geq D_i(x,y) - D_i^- x_i - D_i^+(1-x_i), \ (\text{for } i = 1,2,\ldots,n),$$

$$L(x,y),$$

$$x_i \text{ and } y_i = \{0,1\}, \ (\text{for } i = 1,2,\ldots,n),$$

$$\xi_i \geq 0, \ (\text{for } i = 1,2,\ldots,n),$$

where:

$D_i(x,y)$: a linear function of $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$,
$L(x,y)$: a set of linear constraints.

For example, the satisfaction function associated with the budgetary deviation $\delta_B^+$, that we have presented in section 3 (Fig. 3), can be written as: $F_B^+(\delta_B^+) = \lambda_{B1} f_{B1}(\delta_B^+) + \lambda_{B2} f_{B2}(\delta_B^+)$, where:

$$f_{B1}(\delta_B^+) = \begin{cases} 1 - (\delta_B^+/400000), & \text{if } 0 \leq \delta_B^+ \leq 400000, \\ 0, & \text{otherwise.} \end{cases}$$

$$f_{B2}(\delta_B^+) = 0, \ \text{if } 400000 \leq \delta_B^+ \leq 800000,$$

$$\lambda_{B1} + \lambda_{B2} = 1,$$

$$\lambda_{B1} \text{ and } \lambda_{B2} = \{0,1\}.$$

The deviation can thus be part of one of the two components of the function (the decreasing linear part or the null part). To allow the program to choose one of the two parts of the function, we multiplied each of the two terms of this function by binary variables $\lambda_{B1}$ and $\lambda_{B2} = \{0,1\}$ whose sum was equal to 1. The developed function became:

$$F_B^+(\delta_B^+) = \lambda_{B1}(1 - 0.0000025\delta_B^+) + \lambda_{B2}(0) = \lambda_{B1} - 0.0000025\lambda_{B1}\delta_B^+.$$

We added another constraint to keep the deviation in the intervals limited by the function:

$$0\lambda_{B1} + 400,000\lambda_{B2} \leq \delta_B^+ \leq 400,000\lambda_{B1} + 800,000\lambda_{B2}.$$

We can thus conclude that to reduce the positive deviation of the budget or to seek to maximize satisfaction according to this objective was equivalent to:

$$\text{Max. } Z = \lambda_{B1} - 0.0000025\lambda_{B1}\delta_B^+$$

subject to:

$$400000\lambda_{B2} - \delta_B^+ \leq 0,$$
$$\delta_B^+ - 400000\lambda_{B1} - 800000\lambda_{B2} \leq 0,$$
$$\lambda_{B1} + \lambda_{B2} = 1,$$
$$\lambda_{B1} \text{ and } \lambda_{B2} = \{0, 1\},$$
$$0 \leq \delta_B^+ \leq 800000.$$

The objective function includes a nonlinear term ($D(.) = 0.0000025\lambda_{B1}\delta_B^+$) which thus should have been linearized. The nonlinear term $D(.)$ can be limited as follows: $D^- \leq D(.) \leq D^+$, where:

$D^-$: corresponds to the minimal value which can take the deviation $\delta_B^+$, 0 in this case;
$D^+$: corresponds to the maximum value that can take the deviation $\delta_B^+$ which is: $400000^* 0.0000025 = 1$ in this case.

The term to be linearized was replaced in the objective function by a new continuous variable denoted by $\xi_B$, the objective function became:

$$\text{Max. } Z = \lambda_{B1} - \xi_B$$

subject to:

$$\xi_B \geq 0.0000025\delta_B^+ - (1 - \lambda_{B1}),$$
$$400000\lambda_{B2} - \delta_B^+ \leq 0,$$
$$\delta_B^+ - 400000\lambda_{B1} - 800000\lambda_{B2} \leq 0,$$
$$\lambda_{B1} + \lambda_{B2} = 1,$$
$$\lambda_{B1} \text{ and } \lambda_{B2} = \{0, 1\},$$
$$\xi_B \geq 0,$$
$$0 \leq \delta_B^+ \leq 800000.$$

We proceeded in the same way for all the other satisfaction functions.

## The main program

$$\text{Max. } Z = \lambda_{sn1} - \xi_{sn} + \lambda_{sp1} - \xi_{sp} + \lambda_{B1} - \xi_B + \lambda_{bni1} - \xi_{ni} + \lambda_{bpi1} - \xi_{pi}$$

subject to:

*Constraints on $S^*$:*

$$\sum_{i=1}^{I} \sum_{j=1}^{n_i} \sum_{k=1}^{3} c_k w_i x_{ijk} + \delta_s^- - \delta_s^+ = S^*;$$
$$\xi_{sn} \geq 0.025\delta_s^- - 15(1 - \lambda_{sn1});$$
$$40\lambda_{sn2} \leq \delta_s^-;$$
$$\delta_s^+ \leq 40\lambda_{sn1} + 600\lambda_{sn2};$$
$$\lambda_{sn1} + \lambda_{sn2} = 1 \text{ and } \lambda_{sn1}, \lambda_{sn2} = (0 \text{ ou } 1);$$
$$\xi_{sn} \geq 0;$$
$$0 \leq \delta_s^- \leq \alpha_{sv}^-;$$
$$\xi_{sp} \geq 0.025\delta_s^+ - (1 - \lambda_{sp1});$$
$$40\lambda_{sp2} \leq \delta_s^+;$$
$$\delta_s^+ \leq 40\lambda_{sp1} + 80\lambda_{sp2};$$
$$\lambda_{sp1} + \lambda_{sp2} = 1 \text{ and } \lambda_{sp1}, \lambda_{sp2} = (0 \text{ or } 1);$$
$$\xi_{sp} \geq 0;$$
$$0 \leq \delta_s^- \leq \alpha_{sv}^+;$$

*Constraints on the budget B (with a budget of \$800,000):*

$$\sum_{i=1}^{I} \sum_{j=1}^{n_i} \sum_{k=1}^{3} a_{ijk} x_{ijk} + \sum_{i=1}^{I} p_i y_i - \delta_B^+ = B;$$

$\xi_B \geq 0.0000025\delta_B^+ - (1 - \lambda_{B1});$
$400000\lambda_{B2} \leq \delta_B^+;$
$\delta_B^+ \leq 400000\lambda_{B1} + 800000\lambda_{B2};$
$\lambda_{B1} + \lambda_{B2} = 1;$
$\lambda_{B1}, \lambda_{B2} = (0 \text{ or } 1);$
$\xi_B \geq 0;$
$0 \leq \delta_B^+ \leq \alpha_{Bv}^+;$

*Constraints on the needs of the clients*

$\sum_{j=1}^{ni} x_{ij1} + \sum_{j=1}^{ni} x_{ij3} + y_i + \delta_{bi}^- - \delta_{bi}^+ = b_i;$ (for $i = 1, 2, \ldots, I$)
$\xi_{ni} \geq \frac{1}{\alpha_{il}} \delta_{bi}^- - \frac{\alpha_{iv}}{\alpha_{io}}(1 - \lambda_{bni1});$ (for $i = 1, 2, \ldots, I$)
$\alpha_{io}\lambda_{bni2} \leq \delta_{bi}^-;$ (for $i = 1, 2, \ldots, I$)
$\delta_{bi}^- \leq \alpha_{io}\lambda_{bni1} + \alpha_{iv}\lambda_{bni2}$
$\lambda_{bni1} + \lambda_{bni2} = 1$
$\lambda_{bni1}, \lambda_{bni2} = (0; 1);$ (for $i = 1, 2, \ldots, I$)
$\xi_{ni} \geq 0;$ (for $i = 1, 2, \ldots, I$)
$\xi_{pi} \geq \frac{1}{\alpha_{io}} \delta_{bi}^+ - \frac{\alpha_{iv}}{\alpha_{io}}(1 - \lambda_{bpi1});$ (for $i = 1, 2, \ldots, I$)
$\alpha_{io}\lambda_{bpi2} \leq \delta_{bi}^+;$ (for $i = 1, 2, \ldots, I$)
$\delta_{bi}^+ \leq \alpha_{io}\lambda_{bpi1} + \alpha_{iv}\lambda_{bpi2};$ (for $i = 1, 2, \ldots, I$)
$\lambda_{bpi1} + \lambda_{bpi2} = 1;$
$\lambda_{bpi1}, \lambda_{bpi2} = (0; 1);$
$\xi_{pi} \geq 0;$ (for $i = 1, 2, \ldots, I$)

*Other constraints:*

$x_{ijk} = \{0; 1\}$ (for $i = 1, 2, \ldots, I; j = 1, 2, \ldots, n_i$ and $k = 1, 2$ and 3)
$\sum_{k=1}^{3} = 1$ (for $i = 1, 2, \ldots, I$ and $j = 1, 2, \ldots, n_i$)
$y_i \geq 0$ and integer; (for $i = 1, 2, \ldots, I$)
$0 \leq \delta_{bi}^+ \leq \alpha_{iv}^+$ and $\delta_{bi}^- \leq \alpha_{iv}^-$

*Decision variables:*

$x_{ijk}$; (for $i = 1, 2, \ldots, I; j = 1, 2, \ldots, n_i$ and $k = 1, 2$ and 3)
$y_i$ (for $i = 1, 2, \ldots, I$).

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